# Mathematics 

## Toolkit of

# Graphic Organizers 



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How do I find the zeros for a polynomial function?


What Are the Properties of Auxiliary Lines and Segments that Can Be Drawn in a Triangle?


Graphic Organizer by Dale Graham and Linda Meyer Thomas County Central High School

Thomasville, Ga.

How do you determine the volume of a solid of revolution using the disc method?


Graphic Organizer by Dale Graham and Linda Meyer

## What are the most common ordered pairs on the Unit Circle?




How do you find the surface area (SA) of a


Finally, use the surface area formula SA =

## How do you find the volume of a cylinder?



Use the volume formula V=

## How do you find surface area of a cone?



Finally, use the surface area formula SA =

## How do you find volume of a cone?



## Use the volume formula <br> V=

## How do you find the surface area of a sphere?



## How do you find the volume of a sphere?



How do you find the surface area of a prism?


How do you find the volume of prism?


Use the volume formula
V=

## How do you find the surface area of a pyramid?



Use the surface area formula
SA =

## How do you find the volume of a pyramid?



## How do you use shells to find volume about the x-axis?



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# Math Graphic Organizers for Students with Disabilities 

About the Authors<br>Dr. Paula Maccini, Associate Professor of Special Education, University of Maryland College Park, is currently investigating special education teachers’ use of effective instructional practices in mathematics for secondary students with mild disabilities.<br>Dr. Joseph Gagnon, Assistant Professor, George Mason University, College of Education and Human Development, is currently investigating interventions that positively impact the performance of students with learning disabilities in secondary general education classrooms.

## Introduction

Many school districts and states rely on the Principles and Standards for School Mathematics from the National Council of Teachers of Mathematics (NCTM) (2000). This common reliance (Blank \& Dalkilic, 1992; Parmar \& Cawley, 1995) has led to a greater emphasis on real-world problem solving in the middle school classroom. Complex, open-ended problems require students to identify and use relevant information and to systematically approach problem representation (i.e., organizing the problem-solving process and translating the information to mathematical symbols) and problem solution (i.e., application of appropriate operations to solve a problem) (Maccini \& Ruhl, 2000).

Representing and solving mathematics problems can be particularly difficult for students with learning disabilities (LD). These students commonly have difficulties identifying key information, connecting relationships between broad concepts and details, and strategically approaching the solving of mathematical word problems. Students with LD may also lack fluency with mathematical facts and have problems with basic mathematical procedures (Maccini \& Ruhl, 2000).

Given common adherence to the NCTM Principles and Standards and the noted difficulties experienced by students with LD, it is essential that teachers be informed of and use validated instructional practices (Maccini \& Gagnon, 2000). One method of assisting students is the use of graphic organizers (GOs). In this Issue Brief, we discuss five topics concerning the use of GOs in mathematics for middle school students with LD: (a) definition of graphic organizer, (b) types of GOs, (c) examples of three types of GOs, (d) key components of GOs, and (e) how to use GOs. Finally, we provide two examples of teachers using GOs from secondary general education and resource classrooms.

## What is a Graphic Organizer?

Graphic organizers (GOs) are diagrammatic illustrations used to organize and highlight key content information and/or vocabulary (Lovitt, 1994). Words and/or phrases are used to connect the content information in a meaningful way to help students gain a clearer understanding of the material (Fountas \& Pinnell, 2001, as cited in Baxendrall, 2003). Further, the content is organized with diagrams to help students maintain the information over time (Fountas \& Pinnell, 2001, as cited in Baxendrall, 2003). Research indicates that use of GOs is effective for helping both middle school and secondary students, with and without disabilities, organize and remember content area information (Horton, Lovitt, \& Bergerud, 1990). Additional research indicates use of GOs is also valuable for teaching this group of students, how to represent problem situations in diagrammatic form (i.e., schematic diagrams that address identifying and representing structures or type of word problem) and how to determine the necessary operation(s) needed to find a solution in a problem (Jitendra, 2002).

## What are the types of Grapic Resources?

Common GOs used in mathematics include hierarchical diagrams, sequence charts, and compare and contrast charts (Baxendrall, 2003). Below we provide a definition and example for each GO that is effective for teaching mathematics to middle school students with learning disabilities.

- Hierarchical diagramming involves having a main branch for the overall concept or information, followed by connected sub branches of supporting information or details. The main concept and sub branches are linked by arrows, lines, colors, numbers and/or phrases to show the connections of the hierarchically displayed information. For example, the algebra classroom example below shows a hierarchical graphic display that has the main branch (polynomials) and sub branches or types of polynomials (monomials in red, binomials in blue, and trinomials in green), as well as related examples under each category. Notice that non-examples are incorrect examples of a concept/term that are provided for clarification (e.g., binomial).


## Hierarchical Graphic Organizer

Polynomials

| Monomial <br> (polynomial of one term) | Binomial <br> (polynomial of two terms) | Trinomial <br> (polynomial of three terms) |
| :---: | :---: | :---: |
| 5 | $5 \mathrm{a}+5 \mathrm{~b}$ | $5 \mathrm{a}+6 \mathrm{c}+12 \mathrm{~d}$ |
| x | $10 \mathrm{~h}+10 \mathrm{i}$ | $\mathrm{x} 2+2 \mathrm{x} 2+4 \mathrm{x} 3$ |
| 5 b | $10+12 \mathrm{i}$ | $4 \mathrm{x} 2+3 \mathrm{x} 2+6 \mathrm{x}$ (non-example) |
| $1 / 5$ | $7 \mathrm{y}-2 \mathrm{x}$ | $3+4 \mathrm{x}+\mathrm{x} 2$ |
| $10 / 2$ | $3 \mathrm{x}-4 \mathrm{x}$ (non-example) |  |
| $5 \mathrm{a}+5 \mathrm{a}$ |  |  |

## Teacher: "Write five different examples for each column"

| 1. | 1. | 1. |
| :--- | :--- | :--- |
| 2. | 2. | 2. |
| 3. | 3. | 3. |
| 4. | 4. | 4. |
| 5. | 5. | 5. |

- Sequence charts represent a sequence of events or procedures in a content area. To make the order of the sequence clear, arrows are used that typically flow in one direction (Baxendrall, 2003). It is also recommended that numbers be used for each step to show the flow of events or steps. An example would include diagramming a general problemsolving strategy.


## Sequence Chart

| Polya's Four Problem-Solving Steps |
| :---: | :---: |
| 1. Understand the Problem |
| (What is the goal? Draw a representation ) |
| 2. Devise a Plan |
| (Is there a similar problem I can relate to this? ) |
| 3. Carry out the Plan |
| (Carry out plan and check each step) |
| 4. Look Back |
| (Check answer) |
| Adapted from Polya, 1957 |

- Compare and contrast charts highlight differences and similarities across two or three ideas or sets of information. A common example in mathematics is the use of a Venn diagram. It is important to include ample space on blank diagrams for students to write in responses.


## Compare and Contrast or Venn Diagram

Types of Numbers


## What Are the Components of Graphic Organizers?

In general, teachers should use GOs with students consistently, coherently, and creatively (Baxendrall, 2003).

First, consistent use of GOs within your classroom routine assists students in organizing and retaining information. Similar types of GOs can be used consistently across and within curricular areas for representing information. For example, a sequence diagram can help students represent sequential processes of an algorithm and solve a mathematical problem.

Second, to create coherent GOs, it is important that the displayed information be clear and free of irrelevant information and other distractions. Specific components of coherent GOs include-
(a) clearly labeled branch (i.e., main idea) and sub branches (i.e., supporting detail or steps), and
(b) numbers, arrows, or lines to show the connections or sequence of events/steps between and across ideas or steps.

Because students with LD often have difficulty focusing on important and relevant information (Maccini \& Hughes, 2000), it is particularly critical for teachers to provide direct instruction in how to develop a coherent GO and provide additional supports to students, as necessary. Direct instruction includes these key components:
(a) review
(b) presentation
(c) guided practice
(d) corrections and feedback
(e) independent practice
(f) weekly and monthly reviews (see Rosenshine and Stevens, 1986)

To support students with LD, teachers can also provide a GO that is partially completed and guide students in the process of adding key terms.

Third, it is also important for teachers to find creative approaches that integrate GOs into instruction to heighten student interest. Examples include using small group activities in which group members (or learning pairs) are responsible for filling out parts of a GO and sharing with their group members. Use of cooperative groups or peer tutoring can improve the motivation of students with disabilities and their attitudes toward mathematics (Calhoon \& Fuchs, 2003; Franca \& Kerr, 1990).

## How do I Use Graphic Organizers in My Classes?

Best practice for utilizing GOs in classrooms includes both teacher-directed and student-directed arrangements (Lovitt, 1994). In fact, both approaches are helpful for middle and high school students with LD within general education classrooms (Horton, Lovitt, \& Bergerud, 1990).

- Teacher-Directed Approach: The teacher-directed model includes (1) providing a partially incomplete GO for students (i.e., including the main structures/boxes and interconnecting lines/phrases and omitting information in subordinate areas), (2) having students read the instructional passage or information, (3) providing direct teacher instruction of the information read while referring to an identical teacher copy of the GO on an overhead, (4) filling out the GO with students, (5) reviewing the completed GO, and (6) assessing students using an incomplete copy of the GO.
- Student-Directed Approach: Another effective option is to have students assume more responsibility for completing the GO by using a cover sheet with prompts. For example, the teacher may provide students with a cover sheet that includes page numbers and paragraph numbers to locate the passage information needed to fill out the GO. Students can also be given prompts at the bottom of the page to assist them in identifying key facts and concepts. During the student-directed activity, the teacher acts as facilitator (i.e., circulating the room and monitoring student performance). Prior to studying the completed GO for subsequent assessment, students check their answers with a teacher copy supplied on an overhead.

Teachers can decide to use either approach based on the lesson objectives and the needs of the learners (Horton, Lovitt, \& Bergerud, 1990). For instance, when using the teacher-directed approach, there is an opportunity for the teacher to (a) control the pace of instruction, (b) add more factual or background information during instruction, and (c) encourage group discussions regarding the content. The benefits of a more student-directed approach include greater opportunity to provide (a) individualized teacher assistance to students, and (b) student practice using referential clues or prompts to locate information (Horton et al., 1990).

## How do I Create a Graphic Organizer and What Resources Are

## Available?

Horton and Lovitt (1989 as cited in Lovitt, 1994), recommend a four-step process for developing a graphic organizer. The steps include the following:

- Choose content information that is difficult for students to understand and/or is poorly organized. If choosing information from textbooks, divide the information into about 1500-word segments/passages.
- Develop an outline of the key ideas/concepts in the target information.
- Select an appropriate graphic organizer format that will accurately represent the structure of the content information (e.g., hierarchical, compare and contrast, sequence).

Construct both a completed teacher version of the GO and an incomplete copy of the student version of the GO to use during class instruction.

## Computer Programs for Developing Graphic Organizers

Inspiration Version 7.6 by Inspiration Software, Inc. Portland, Oregon. The program offers software to help educators individualize instruction for learners in grades 6 and higher. The graphic tools help teachers create a variety of organizational devices, such as concept diagrams, webs, outlines, and maps. The company Web site is: www.inspiration.com

Mind Mapping Software by the Buzan Organization Ltd, Palm Beach, Florida. The program offers software to help educators customize lessons, presentations, and handouts. The software can be used to create organizational diagrams. The company website is: www.nova-mind.com

## Examples of Graphic Organizers in Secondary Mathematics Classes

Two examples of the use of GOs are illustrated below, including the three components (coherence, consistency, and creativity), in a secondary mathematics resource room and general education classroom. The examples emanated from case studies of secondary special education mathematics teachers.

## Classroom Example 1: Resource

Classroom
The secondary mathematics resource classroom consisted of 10 students, mostly 9 th grade students with LD, in an applied mathematics class.

Lesson Objective : The lesson objective addressed a review of measurement and writing fractions in simplest form. The teacher used a sequence chart GO (i.e., a constructed number line) to help students visualize how to represent equivalent fractions and how to simplify fractions to simplest terms.

Materials : Ruler, paper, different colored pens, various objects to measure.

## Instructional Sequence:

- To help students understand how to create a coherent sequence chart GO, the teacher models how to construct a number line on an overhead. The teacher uses different colored pens to denote each fractional part. The sequence or steps the teacher uses to create the chart include-
- using a red marker to note halves (appears in blocks),
- using a blue marker to note fourths (appears in bold),
- using a green marker to note eighths (appears as dashes), and
- using a black marker to note sixteenths (appears as dots)
- The teacher provides guided practice and monitors students as they construct a number line with their rulers.
- To check for accuracy, the teacher has students compare their number lines with the markings on their individual rulers.
- The teacher then models, using examples, how to use the constructed number line to reduce fractions to simplest form: $8 / 16=4 / 8=2 / 4=1 / 2$ (see graphic below).
- Students then measure the length of sides of various geometric objects with their number lines. First they measure the side of a triangle, the long side of a parallelogram, and a square. Then, they use the constructed number lines to help them simplify measurements to simplest form. For creativity in use, students can work together to measure objects using the constructed number lines.
- For consistency of use, the GO can be reviewed in future lessons to help students represent fractions and to reduce to simplest form.

| 0 |  |  |  |  |  |  |  | $1 / 2$ |  |  |  |  |  |  |  | $2 / 2$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  | $1 / 4$ |  |  |  | $2 / 4$ |  |  |  | $3 / 4$ |  |  |  | $4 / 4$ |
| 0 |  | $1 / 8$ |  | $2 / 8$ |  | $3 / 8$ |  | $4 / 8$ |  | $5 / 8$ |  | $6 / 8$ |  | $7 / 8$ |  | $8 / 8$ |
| 0 | $1 / 16$ | $2 / 16$ | $3 / 16$ | $4 / 16$ | $5 / 16$ | $6 / 16$ | $7 / 16$ | $8 / 16$ | $9 / 16$ | $10 / 16$ | $11 / 16$ | $12 / 16$ | $13 / 16$ | $14 / 16$ | $15 / 16$ | $16 / 16$ |

## Example 2: Algebra, Mrs. Lesma Forrester's Algebra Class

A secondary mathematics teacher used a GO in her general education algebra class. The class consisted of 20 students and included students with LD. When introducing the term "polynomial," she used a GO to categorize the three types of polynomials.

Lesson Objective : Introduce the term polynomial and different types (monomial, binomial, and trinomial).

Materials: Yellow colored paper for making graphic organizer (student copy); one overhead version of GO for the teacher.

## Instructional Sequence:

- Following the daily review, the teacher provides a piece of yellow paper to each student and mentions they are going to work on polynomials. For consistency, the teacher uses a color-coded sheet to help students quickly access the GO in their mathematics binders for use throughout the term. Students can quickly locate examples of each type of polynomial by looking at their completed GO.
- The teacher models folding the paper into three columns and students also complete the task. To make the GO coherent, the teacher minimizes distractions on the GO (e.g., clear heading, labeled subheadings, adequate space provided for additional student-generated examples).
- The teacher provides a class discussion (e.g., "Does anybody have an idea of what the prefix 'poly' means?"). Following the discussion, the teacher asks students to write "something that represents one quantity." The teacher models in two ways: with examples of terms representing one quantity (e.g., $5,3 \mathrm{a}, \mathrm{x}, 1 / 5,10 / 2$ ) and with examples and non-examples that differ by one attribute (e.g., $5 a+5 b$ and $5 a+5 a$ ).
- The teacher provides guided practice and has students examine the examples for similarities. The teacher prompts students to think of a name for quantities that represent one quantity or term. The term monomial is defined at the top of the first column (see previous example of a hierarchical graphic organizer).
- The teacher probes students to determine the name for the second column (e.g., "If this one is monomial, what, do you think, this one is going to be called?", "What is the prefix for two?") and the definition of terms ("This polynomial that has two terms is called a binomial. The polynomial with one term is called a monomial"). The terms are written at the top of each column in the GO, followed by additional teacher modeling of examples and non-examples of binomials and trinomials.
- For independent practice, students are asked to use their GO and write down five additional examples under each of the three categories. For creativity in use, students work together to generate ideas as the teacher circulates and monitors.


## Conclusions

Rather than a focus on basic skills instruction in isolation, the instructional focus supported by the NCTM Principles and Standards for School Mathematics (2000) includes teaching higherlevel thinking, reasoning, and problem-solving skills that are embedded in real-world situations (Gagnon \& Maccini, 2005). While this approach gives greater meaning to mathematics tasks, it also results in challenges for students with LD. These students require additional support to focus on relevant information and organize their approach to solving mathematical problems. GOs are one valuable tool for assisting middle school students with LD with basic mathematical procedures and mathematical problem solving. To effectively use GOs, teachers should-
(a) consistently, coherently, and creatively use GOs;
(b) employ teacher-directed and student-directed approaches; and
(c) address individual needs via curricular adaptations (e.g., provide partially completed GOs, highlight information in the text, provide cues at the bottom of a blank GO, provide group
activities). Provided these critical features are included, GOs have great potential as a mathematics strategy for middle school students with LD.

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# How do you fine area between curves? ${ }^{29}$ 



How do you graph linear equations using the intercepts?


# How do you use amplitude and peri3d to graph sine and cosine? 

## Amplitude Change

Graph $\mathrm{y}=2 \sin \mathrm{x}$

| x | $\sin \mathrm{x}$ | $2 \sin \mathrm{x}$ |
| :---: | :---: | :---: |
| 0 |  |  |
| $\frac{\pi}{2}$ |  |  |
| $\pi$ |  |  |
| $\frac{3 \pi}{2}$ |  |  |
| $2 \pi$ |  |  |

Your Turn. Graph $y=-\frac{1}{2} \cos x$


## Period Change

Graph $y=\cos \frac{x}{2}$

| x | $\frac{x}{2}$ | $\cos \frac{x}{2}$ |
| :---: | :---: | :---: |
| 0 |  |  |
| $\pi$ |  |  |
| $2 \pi$ |  |  |
| $3 \pi$ |  |  |
| $4 \pi$ |  |  |



What is the period of this graph? $\qquad$

Your Turn. Graph $y=\sin 2 x$
What is the period of this graph?

How do you graph transformations?
Equation Transformation
Graph

Name: $\qquad$

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## How do you solve a system of equations by graphing?



| $2^{\text {nd }}$ |
| :---: |
| Graph the lines. |
| $3^{\text {rd }}$ |
| Look for the intersection |
| and give your answer. |
| Same line? |
| Infinite solutions |
| Parallel lines? |
| No solution |
| Intersecting lines? |
| Name the intersection |




Median

Angle Bisectors

- bisect angles
- concurrent at incenter



Median

How do you find the slope of a line given two points on the line?


## TOPIC:

$\qquad$

Name: $\qquad$ Date: $\qquad$

Common Factors


Find Common Factors
of $\qquad$ and $\qquad$


Find Common Factors of $\qquad$ and $\qquad$


## How do you use disks to find volume about the x-axis?



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$c^{2}=a^{2}+b^{2}$

When $c$ is unknown:
When $a$ or $b$ is unknown:

## How do you find the measure of various angles associated with parts of a circle?

## Determine the location of the

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| Main Math Concept <br> and Vocabulary | Notes on Steps to Solve Model Problem | Why does this work? |
| :--- | :--- | :--- |
|  |  |  |

Notes for: (Type of Math or Math Skill Set)

| Problem and How to Do it: Label problem with <br> proper math vocabulary. | Rules or steps for this type of problem: |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

Problem and How to Do it: Label problem with $\quad$ Rules or steps for this type of problem: proper math vocabulary.

## Understanding Algebraic Expressions

| Algebraic Expression <br> (List it and Solve it) | Rationale | Property, Theorem, or <br> Postulate |
| :--- | :--- | :--- |
| $\mathrm{c}(\mathrm{a}+\mathrm{b})$ | Order of operations and variables on <br> numeral outside the ( ) | Distributive Property |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Make up your own real world problem using one of the algebraic expressions above. Work the problem. Then explain why this expression is important in terms of real world applications of this type of math. Use at least two authenticated websites to substantiate the real world importance of this expression. Use the back as needed.

