## Rigorous and

## Relevant Literacy

## Part III

# With Lin Kuzmich Senior Consultant, ICLE Port Huron Area School District High School March 2012 

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## Welcome!

- Suffering is Optional
- Participation is Most Appreciated
- Electronics on vibrate
- Getting Your Questions Answered
- Participation Appreciated
- Computer use for note taking is fine, however please check your email only during breaks or lunch
- Listen with the Intent to Understand


High Payoff Literacy Strategies are in these Categories: The Big 8


1. Vocabulary
2. Student Dialogue and Grouping
3. Write to Learn
4. Graphic Organizers and Note Takers
5. Teacher and Student Questioning
6. Document, Technological, and Quantitative Literacy Strategies
7. Leveled Materials and Digital, Multi-Media Resources
8. Text and Media Complexity Access

## 1. Introduction



Introduction: Reading Tasks Specific to Your Content Area


What text based comprehension skills do students need to be successful in your content area or course?

What document skills (numerical array, picture, video, chart, map, graph, table) do students need to be successful comprehending or using in your content area or course?

1. Write a few words to remind you of the most important thing you learned in the clip. Important to you or your situation.
2. Draw a picture or symbol to remind you of the important idea you described in \#1.
3. Write a question that could be answered by watching this video clip.
4. Write a second question at analysis, evaluation or creation levels. (For math you could create a word problem)

What do you infer?

| What is your <br> inference? | What is your proof? |
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| Inference Proof Notes |  |
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| Your <br> Inference Your Proof <br> Students are <br> learning <br> something on <br> computers Computers in front of students, <br> man gesturing in the direction of <br> the screen like a teacher might <br> These pictures <br> are about <br> shapes Each picture contains parts of a <br> circle |  |

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Video Snapshot:
A Four Corner Foldable for Increasing Rigor in Technological or Media Based Literacy

| 1. Write a few words to <br> remind you of the most <br> important thing you <br> learned in the clip. <br> Important to you or your <br> situation. | 2. Draw a picture or <br> symbol to remind you of <br> the important idea you <br> described in \#1. |
| :--- | :--- |
| 3. Write a question that <br> could be answered by <br> watching this video clip. | 4. Write a second question <br> at analysis, evaluation or <br> creation levels. (For math <br> you could create a word <br> problem) |



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Four Prerequisites for Inferential Thinking Math or Science $=90 \%$ Improvement in Inference

| $\frac{\text { 1. Big Ideas - } 1 \text { to } 2 \text { for Math or }}{\text { Science }}$ 2 Example, Hypothesis or Model <br> Problem for Math/Science <br> 3. Analysis - Relevance, <br> Usefulness, or Importance of the <br> Information (May also be Rules, <br> Characteristics, Attributes, or <br> Patterns - for math and science) <br> 4. Evaluate the best method, <br> strategy or experimental design to <br> answer the question or <br> prove/disprove the hypothesis. <br> Inference Question and Answer: (includes words like - what if, implied <br> by, opinion, estimate, guess, prediction, deduction, assumption, <br> implication)  |
| :--- | :--- |

## Other Ways to Build Inferential Capability in Math, Science, and Technical Course Using Math or Science

- Observation: of phenomena or process, what do you see or know, why, how would you describe this
- Generalization: what are the rules, patterns, sequence, attributes or characteristics that govern this
- Justification: Formulate a hypothesis, solution, prediction, estimation and justify it based on your observations and generalizations. Then prove whether you are right or wrong, analyze the data, calculation or outcome to prove or disprove your original justification.
Based on the research of: The National Council on Improving Student Achievement in Math and Science and the University of Wisconsin

Scientific and Mathematical Learning that Increases Long Term Memory and Rigorous Thinking

| Observation and <br> Model Building | - look like, function like, descriptive, <br> explanatory, build models |
| :--- | :--- |
| Generalization | synthesis of data or observations using <br> reasoning and logic, create rules or criteria |


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## 4. Student Dialogue: Jigsaw Learning

| - Brain bits: <br> - Dialogue increases memory <br> - Students needs processing break every 7 to 12 minutes from age 12 on, less time between for younger students and those who are behind, confused, or struggling <br> - The person doing the work is doing the learning | - Think-Pair-Share helps improve writing, and therefore thinking skills <br> Jigsaw Methods help Students do more of the work in initial learning and strengthens both buy-in and further learning |
| :---: | :---: |
|  |  |

- Traditional or typical Cooperative Group Strategies are not enough
- Cooperative Group

Strategies are a
starting point in dialogue and grouping

- Need study skills
based strategies and
flexible grouping for a diverse population that may be underperforming...
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## Dialogue, Grouping and Jigsaw for Better Critical Thinking

Stages and Scaffolding

- Think-Pair-Share
- Small Group Jigsaw
- Study Groups
- Study Buddy -

Elementary
Study Team - Middle Level

- Study Teams - High School

Tips and Tricks that Work

- Think-Pair-Share
- Use Q and A cards
- Role Play first
- Small Group Jigsaw
- Simple
- Table Top
- Study Groups
- Roles matter
- Divide up work
- Use first for test review


## Three Types of Jigsaws

- Simple Jigsaw: You may use a simple square jigsaw of 3, 4, or 5 people. Each person must become knowledgeable about one piece of information in a table group and then teach the new information to the rest of the group.
- Expert Jigsaw: Similar to the Simple Jigsaw except each expert from all the groups will join together with other experts to discuss the material before teaching it back in the original grouping.
- Table Jigsaw: Each table is responsible for a different piece of content. Each table studies the material and decides how to teach it to the large group.

Jigsaw Works With Diverse Learners

Types

- Expert group traditional jigsaw
- Table top jigsaw
- Partners jigsaw

Works great given highly diverse students with varying background knowledge and skills

3. Five Text Coding and Questioning Methods for Tough Passages/Items and for Struggling Learners
A. Focused - on words and big ideas
B. Layered - on elements of thinking
C. VIP Coding - especially good for materials that cannot be written in or upon like text books, but can be used with any text or figures
D. KISSR Method of Coding Problems- for math or any item containing math in science, CTE, or social science
E. DTQ Questioning/Strategies- for charts, maps, graphs, tables, and other array based representation of information that is not in paragraph form

Kuzmich, 2010

## A. Focused Coding for Key Vocabulary

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Steps for Focused Coding to identify math vocabulary in an explanation or word problem:

1. Selected number of content words to find and have students highlight or circle the words
2. Have students justify what they highlighted or circled with a partner
3. Students develop their own definition or rationale for the importance of the words

## B. Layered Coding - Expanding Question Based Coding of Text

## Layered Coding

1. Question the text (Yourself)

What is the Purpose or the Importance of the text?
What is the Big Idea or Content Focus?
What is the Question at Issue or Point of View?
2. Predict answers

What are your assumptions?
What is your evidence or information that supports your assumption?
3. Share with others
4. Draw defensible conclusions, cite implication or cite consequences using text vocabulary and quotes share verbally with your partner
5. Now write a short paragraph* (*optional or could be your Quick Write or assessment for the day)

Kuzmich, 2010
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## Text Coding Methods

## Created by Lin Kuzmich, 2009

Text coding helps students derive higher levels of meaning from text. These two methods work well during reading and to prompt re-reading of any type of text.

## Focused Text Coding:

1. Select a number from 3 to 6 of words or phrases to highlight with a clear focus
2. Select a personalized or relevant prompt
3. Ask that prompt at the analysis, evaluation or synthesis level
4. Read or re-read the entire selection or portion of text, stopping to coding during the reading process

## Layered Text Coding:

1. Read or re-read just a section of text
2. Write a question for the author
3. Speculate on the answer to the question and justify why you arrived at that possible answer
4. Complete this after reading or re-reading the selection

Additional Notes and Suggestions: These two methods can easily be combined. Focused text coding can be used either for initial reading or for re-reading text. By adding personal relevancy and keeping the prompt questions at analysis, synthesis and evaluation, even a simple reading for information activity can become a Quadrant D piece of the lesson. Add an opportunity to share responses, questions and words or phrases highlighted and you add even more value to the lesson. End with a Quick Write to pull it together and you have an easy mini-assessment or check for understanding. These two methods are highly recommended for struggling learners to learn prior to more complex methods.

## B. Layered Coding - Expanding Question Based Coding of Problems in Math or Science <br> Layered Coding <br> 1. Develop a question about a problem, data set or explanation (Yourself) <br> 2. Predict answers to the question with a partner or <br> 3. Trade questions and answer them or ask the question in whole group sharing <br> 4. Optional: In a Quick Write answer one of your questions

## Rules for Layered Text Coding in Math

- Code more than content: thinking words, rules, sequence, comparisons, directions, opinions, etc.
- Develop rules, sequence or summary statement or questions


## C. VIP Coding

Text with Symbols or
Highlighting -
Reminders

- Highlighting
- Use of color
- Circle
- Underline
- Check or question mark
- Other markings to help decode meaning


Kuzmich, 2010

VIP Coding - Works in Text Books When You Can't Write in the Book or on the Materials

```
        Coding Key Ideas:
v.I.P. Very important
information or purpose, key
concepts
? Information you are
    confused about or doesn't
    make sense
! Wow or Aha, something you
    find interesting, hard to
    believe, or unexpected
\checkmark Something you already knew that is
    important for learning
# something that must be done in
    sequence
```



## Directions for VIP Text Coding

## As you read the text...

Use your 8 post-it-note "markers" to mark sections of your text that you find to be key concepts or information (V.I.P.), or information you're confused about or that doesn't make sense (?), or information you find interesting, hard to believe, was unexpected (!), critical information you already know $(\sqrt{ })$ or important sequence (\#).
You may find that you need to
move your markers as you read new information. Your goal is to use all eight markers.
When you finish reading . . .
Go back to each marker and place the appropriate symbol on each marker.

For every V.I.P summarize the ormation in your own words
every? write a question to
doesn't make sense doesn't make sens
For every ! summarize what/ why you
found this interesting.
For every $\sqrt{ }$ justify why this already known information will be key in our learning of this skill, understanding this text, or needed
to arcomplish a task such as problem solving or writing or a
lab.
For every \# write the number of the
rule or sequence rule or sequence on your flag and
then write out the rule or step in your notes

Note: keep these markers in your text; they will be helpful when you Wen you finish your
When you finish your markers Transfer the information from your markers onto the Coding Text
Worksheet - we will use this again.
We will beruishing ithis information at ${ }_{4}$

## D. Math, Social Science, CTE, and Science Tip: KISSR Coding for Math Problems

Keywords - highlight or underline key math words
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Identify the question being asked - what you are to find or do -circle the part of the question $\qquad$
Strategy - choose a strategy to help you (i.e. draw a picture, make a chart or graph, work $\qquad$ backwards, guess and test, etc.)
Solve the problem
Review and revise (check your answer, is your answer reasonable)

Math Department at South Houston High School in Texas 2008

## Redefining Literacy in Grades 7-12

The model in Figure 6 shows the three areas of essential competencies in document, technological, and quantitative literacies. Within these three areas are 14 skills that every student needs to demonstrate literacy with documents, technological sources, and documents requiring quantitative solutions.

Figure 6
Three Competency Areas for DTQ Literacy


## E. DTQ Methods

- For charts, maps, graphs, tables, and other documents, visuals, quantitative (math problems), or materials that are not in paragraph form
- DTQ = Document, Technological, and Quantitative Literacy




## DTQ Teaching Tips

- Finish data or information on a chart, table, graph, picture, website, or map
- Organize information into a chart, table, graph, picture, website or map
- Authenticate and justify an electronic source for information or opinions
- Write questions that could be answered by the chart, table, graph, picture, website, or map
- Practice answering comprehension, analysis, summary, and evaluative questions using a chart, table, graph, picture, website, or map

Kuzmich, 2010 table, graph, picture, website, or map
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Finish the Next Data Point for this Chart

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## Finish this Math Problem

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\begin{aligned}
& \text { Solve } x^{2}+3 x-4=0 \\
& \text { This quadratic happens to factor: } x^{2}+3 x-4=(x+4)(x-1)=0 \\
& \text {..so I already know that the solutions are } x=-4 \text { and } x=1 . \\
& \text { How would my solution look in the Quadratic Formula? Using a } \\
& =1, b=3 \text {, and } c=-4 \text {, my solution looks like this: } \\
& \qquad x=\frac{-(3) \pm \sqrt{(3)^{2}-4(1)(-4)}}{2(1)} \\
& \qquad=\frac{-3 \pm \sqrt{9+16}}{2}=\frac{-3 \pm \sqrt{25}}{2} \\
& \text { Complete the } \\
& \text { Missing Step } \quad \begin{array}{l}
=\frac{-8}{2}, \frac{2}{2}=-4,1=x \\
\text { Then, as expected, the solution is } \boldsymbol{x}=-4, \boldsymbol{x}=1 .
\end{array} .
\end{aligned}
$$

| Why can't this problem be solved? <br> Look at the following chart to determine the average percentage of students who feel unsafe during passing periods. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Number of Students Reporting Feeling Unsafe |  |  |  |  |
|  | $9^{\text {th }}$ Grade | $10^{\text {th }}$ Grade | $11^{\text {th }}$ Grade | $\begin{aligned} & 12^{\text {th }} \\ & \text { Grade } \end{aligned}$ |
| Before school | 25 | 35 | 15 | 10 |
| Between classes | 65 | 42 | 37 | 18 |
| At lunch | 51 | 22 | 10 | 4 |
| After school | 62 | 53 | 19 | 26 |
| At events | 38 | 27 | 22 | 12 |
| ${ }_{5}$ Coprright Kuzmich, 2007 |  |  |  |  |

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## Develop Your Own Questions




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## Analytic Thinking for Text or Information Sources from Elder and Paul, 2003, p. 10

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1. What is the purpose of this material?
2. What is a key question that is addressed or needs to be addressed?
3. What is the most important information?
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4. What are the main inferences that can be made? $\qquad$
5. What are the key ideas or concepts?
6. What are the assumptions the author(s) made in this information, issue, or source? $\qquad$
7. What are the implications of this information?
8. What is the main point of view that is $\qquad$ presented?


| Practical Use of Thinking Standards |
| :--- | :--- |
|  Math-Specific Questions or Prompts <br> Clarity Andre, can you explain Latisha's solution strategy in other words? <br> Precision Could you be more specific about the answer? What does it represent? <br> Accuracy How can you assure us that the information you are using is accurate? <br> Relevance How can you use what you've learned from this problem in your life? <br> Depth What questions do you have about this topic due to your work on this <br> problem? <br> Breadth What are some other ways you could have solved this problem? <br> Logic Can you explain the logic (sequence) of your thought process when <br> solving this problem? <br> Significance What is the most important aspect of this problem and the solution? <br> Adapted from: Paul, R. \& Elder, L. (2007). How to Improve Student Learning. <br> Foundation for Critical Thinking Press.  |

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Adapted from: Paul, R. \& Elder, L. (2007). How to Improve Student Learning.
$\qquad$ Foundation for Critical Thinking Press. $\qquad$

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## ABC Reading and Student Questioning Protocol

1. Put students into groups of three making certain that no group has three struggling readers, if possible
2. Student A reads first paragraph or section
3. Student B puts the passage in their own words
4. Student C asks Critical Thinking Questions (You tell them which two or three types to focus upon)
5. Then rotate tasks until all have a turn at each task or text is completed.

## B. When Cause and Effect Thinking Gets More Complex Try Root Cause Analysis

## Two Types of Causes

- Conditional Causes - multiple conditions exist in the data gathered, two or more variable exist
- Cause analysis must look at each condition or variable separately and an analysis of how they interact
- Action Causes - actions that take place upon the conditions or variables and we must analyze how this impacts the effect
- Action Analysis must look at each action separately and then analyze how the actions impact the result

1. Create a table to do this type of analysis
2. Always write your conclusion and justify it.

Note: you can use this approach for independent and dependent variables as well.

## Action versus Condition Causes

| In an accident report, we see: |  |  |
| :--- | :--- | :--- |
| Type of Problem <br> (Effect) | What happened <br> (action cause) | Conditions (conditional causes) |
| Injury | Fall | Wet Surface |
| Illness | Vomiting | Tainted milk in lunchroom |


| In an maintenance report, we see: |  |  |
| :--- | :--- | :--- |
| Type of Problem <br> (Effect) | What happened <br> (action cause) | Conditions (conditional causes) |
| Leaky valve | Seal failure | No maintenance |
| Engine failure | Parts seized | Oil was not checked frequently |

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| Real World Examples Help Us Teach Integration of Information |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Effect | Conditions | Actions | Data <br> Source | Integration |
| Car Accident | Moving Truck <br> (1) | Moving truck ahead of accident car | Driver of <br> Truck (1), witness | Did not do anything to contribute one way or another to accident |
|  | Truck Swerved <br> (2) | Moving truck in opposite direction swerved into car | Driver of Truck (2), 3 witnesses | Driver turned steering wheel to avoid childtruck went into path of car causing the accident |
|  | Car in the Way | Car parked on shoulder on passenger side of accident car | Verbal statement by accident victim | Parked on shouldernot the primary cause, but accident car had no where to go |
| Source: Apollo Associated Services, 2006 and Latino, 2006. |  |  |  |  |
| ${ }^{68}$ |  |  |  |  |

## 5. Test Preparation Strategies

There are a few
strategies that have very high payoff

- Many of you asked for these prior to our next visit
- Please review the article in your handout packet and let me know if you have additional questions
Do all three listed strategies
and you increase the
probability of student
success on any single item by
$75 \%$ to $85 \%$ over not using the strategies

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## 3 High Payoff Test Prep Strategies

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1. Leverage your
relationships to
encourage students to
encourage students to
show best efforts in a
systematic way
2. Ask students to name or title each question as to the primary content
being assessed
3. Ask students to tell you which two multiple
choice answers are not
correct and why, pick
from the remaining two and justify the answer selected
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Kuzmich, 2011 $\qquad$

## Content Sort

- Cut up release items and pass out to students
- Pair students up
- Help each other decide what the topic or content that items is testing: linear equations, cell division, grammar, etc.
- Students develop a study guide for that question on a large note card and paste, tape or staple the item on back


## Multiple Choice Formats ELA and Science

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1. Which of these statements from the
advertisement is most likely

intended to encourage the \begin{tabular}{l}
3. Which action may increase the <br>
impact of the myxoma virus on the <br>
rabbit population today? <br>
consumer to act immediately?

 

A. spraying to kill mosquitoes
\end{tabular}

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C. "Order one of our many classic sandwiches. .." rabbit population is low deliver the best to your school."

Which action may inclease the rabbit population today?
A. spraying to kill mosquitoes virus
C. introducing sterile rabbits into the population


## 6. Next Steps


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Rigor and Relevance Quadrant C or D

|  | Rigor and Relevance Quadrant A or B |  |  | Rigor and Relevance Quadrant C or D |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Remembering | Understanding | Applying | Analyzing | Evaluating | Creating |
|  | Choose <br> Describe <br> Define <br> Find <br> Identify <br> Label <br> List <br> Locate <br> Match <br> Memorize <br> Name <br> Omit <br> recall <br> Recite <br> Recognize <br> Reproduce <br> Sequence-simple <br> Select <br> State <br> Tell | Calculate Classify <br> Demonstrate <br> Distinguish <br> Explain <br> Extend <br> Give example <br> Illustrate <br> Indicate <br> Interrelate <br> Interpret <br> Infer <br> Judge <br> Match <br> Paraphrase <br> Represent <br> Restate <br> Rewrite <br> Show <br> Summarize | Apply <br> Chose <br> Dramatize <br> Explain <br> Generalize <br> Judge <br> Organize <br> Paint <br> Prepare <br> Problem- <br> calculation <br> Produce <br> Select <br> Sequence- <br> complex <br> Show <br> Sketch <br> Solve <br> Use | Analyze <br> Categorize <br> Cause/Effect <br> Classify <br> Compare <br> Complex-Infer <br> Contrast <br> Deduce <br> Differentiate <br> Distinguish <br> Identify <br> Point out <br> Organize <br> Select Subdivide <br> Survey | Appraise <br> Argue <br> Estimate <br> Evaluate <br> Determine- bias <br> Judge <br> Criticize <br> Critique <br> Debate <br> Defend <br> Justify <br> Prioritize <br> Pros and Cons <br> Rate <br> Value <br> Verify <br> Weigh | Add to <br> Combine <br> Compose <br> Construct <br> Create <br> Design <br> Develop <br> Devise <br> Forecast <br> Formulate <br> Predict <br> Solution <br> Formulate <br> Hypothesize Invent <br> Originate |
| Instructional Strategies | Highlighting <br> Rehearsal <br> Memorizing <br> Mnemonics <br> Make a list of the main events <br> Make a timeline of events <br> Make a facts chart Write a list of any pieces of information you can remember List all the .. in the text or video | Key examples <br> Emphasize <br> connections <br> Elaborate concepts <br> Summarize - written <br> Paraphrase - oral <br> Students explain <br> Students state the rule <br> Why does this <br> example...? <br> Visual representations <br> Concept maps <br> Outlines- verbal <br> Flow chars <br> Graphic Organizers | Students modeling <br> Sequencing <br> Real World application <br> opportunities <br> Case studies <br> Simulations <br> Algorithms in problem form <br> Construct a model, diorama, scrapbook, to explain ideas or information in context Organize a collection to make a point Make up a game Write an outline | Analogies <br> Models of thinking <br> Challenging <br> assumptions <br> Retrospective analysis <br> Reflection through journaling <br> Collaborative learning <br> like jigsaws <br> Design a <br> questionnaire to gather information <br> Construct a data chart Write a commercial to sell something Conduct an investigation to support a point of view or hypothesis | Challenging <br> assumptions <br> Evaluative journaling <br> Debates <br> Collaboration to <br> evaluate point or view, <br> worth or other specific <br> aspect <br> Decision-making <br> Solution development <br> Problem based <br> learning <br> Prepare a list of criteria to judge.. <br> Self evaluate and then correct your work <br> Prioritize and rationale <br> Justification in writing or verbally <br> Critique art or music or theater - any art form | Design activities Inventions <br> Creation of a model of a solution to a complex problem Devise a new way to solve a complex problem <br> Compose music Create original art |
|  | Who? <br> Where? <br> Which one? <br> What? <br> How? <br> What is the best one? <br> How much? <br> When? <br> What does it mean? | State in your own words <br> Which are facts? What does this mean? Is this the same as... Give an example Condense this paragraph or paraphrase it Explain what is happening What are they saying? What seems to be...? What seems likely? What is the main idea? | Predict what would happen if... <br> Choose the best statements that apply What would result Tell what would happen <br> Tell how, when, where and why <br> Tell how much change there would be Identify the results of... <br> What is the function <br> of...? | Is this fact or opinion Why? <br> What are the assumptions behind...? <br> What is the relevance? <br> What is the motive? <br> What are the conclusions based upon? <br> What does the author believe or assume? Make a distinction State the point of view, rule or pattern What is the relationship between..? | Is it valid that? <br> Judge the effects What fallacies, consistencies, inconsistencies appear or exist? Which is more important, moral, better, logical, valid, or appropriate? Find the errors Defend your point of view or this viewpoint or answer Justify your answer or response or point of view <br> Is bias, fairness, or ethics at issue? | How would you test...? <br> Propose an alternative. Develop a creative solution for... Invent a new process, system, procedure or product that addresses... |
| $\begin{aligned} & \text { 品 } \\ & 0.0 \end{aligned}$ | Level 1: Lit summariz | right there, emember | Level 2: Sear connect and | nd think, infer, , or associate | Level 3: Log derive, or | justification, your own |

Sources: Bloom (1954 and 1956) revised by Anderson and Krathwohl (2001), Dalton, J. and Smith, D. (1986), Kuzmich, L. (2011), Costa , A. and Kallick, B. (2000) and Daggett, W. ICLE Rigor and Relevance Model. Original chart created by Lead4Ward, Region XIII, TX

## Costa's Levels of Thinking and Questioning

Level 1
Remember:
Define
Repeat
Name
List
State
Describe
Recall
Memorize
Label
Match
Identify
Record
Show
Understanding:
Give examples
Restate
Discuss
Express
Rewrite
Recognize
Explain
Report
Review
Locate
Find
Paraphrase
Tell
Extend
Summarize
Generalize

Level 2
Use Understanding:
Dramatize
Practice
Operate
Imply
Apply
Use
Compute
Schedule
Relate
Illustrate
Translate
Change
Pretend
Discover
Solve
Interpret
Prepare
Demonstrate
Infer
Examine:
Diagram
Distinguish
Compare
Contrast
Divide
Question
Inventory
Categorize
Outline
Debate
Analyze
Differentiate
Select
Separate
Point out
Criticize
Experiment
Break down
Discriminate

## Create:

Compose
Design
Propose
Combine
Construct
Draw
Arrange
Formulate
Organize
Compile
Revise
Write
Devise
Modify
Assemble
Prepare
Generate
Level 3
Decide:
Judge
Value
Predict
Evaluate
Rate
Justify
Decide
Measure
Choose
Assess
Select
Estimate
Conclude
Summarize

## Supportive

Evidence:
Prove your answer.
Support your answer.
Give reasons for your answer.
Explain your answer.
Why or why not?
Why do you feel that way?

## Costa's Levels of Thinking and Questioning

## Steps to the Inquiry <br> Process

Higher-level questions are essential to facilitating conceptual understanding. The inquiry process is facilitated by skillful questioning and provides students with the opportunity to become independent thinkers who master their own learning.

STEPS Description of
Inquiry Level Inquiry Level
Sample Questions
Step 1:
Gather and Recall
Information
(Gathering/Input)
Ask Level 1 questions
to identify what students know about the problem or question and connect to prior
knowledge.
-What do you know about your problem?
-What does
mean?
-What did you record from your class notes about the lecture?
-What does it say in the text about this topic?
-What is the formula or mnemonic device (ex. P-E-M-D-A-S) that will help you identify the steps necessary to solve the problem?

Step 2:
Make Sense Out of Information Gathered (Processing)

Ask Level 2 questions to begin processing the information gathered, make connections and create relationships.
-Can you break down the problem into smaller parts? What would the parts be?
-How can you organize the information?
-What can you infer
from what you read?
-Can you find a
problem/question similar to this in the textbook to use as an example?
-What is the relationship between $\qquad$ and $\qquad$ ?

Step 3:
Apply and Evaluate
Actions/
Solutions
(Applying/Output)
Ask Level 3 questions to apply knowledge acquired and connections made to predict, judge, hypothesize or evaluate.
-How do you know the solution is correct? How could you check your answer?
-Is there more than one way to solve the problem? Could there be other correct answers?
-Can you make a model of a new or different way to share the information?
-How do you interpret the message of the text?
-Is there a real life situation where this can be applied or used?
-Can you explain it in a different way?
-Could the method of solving this problem work for other problems?
-How would you teach this to a friend?

## Costa's Levels of Thinking and Questioning

Costa's Content<br>Specific Questions

## Costa's Levels of Questioning Math

Level 1
What information is given?
What are you being asked to find?
What formula would you use in this
problem?
What does
$\qquad$ mean?
What is the formula
for...?
List the...
Name the...
Where did...?
What is...?
When did...
Explain the concept
of...
Give me an example of...
Describe in your own words what $\qquad$ means
What mathematical concepts does this problem connect to?
Draw a diagram of...
Illustrate how $\qquad$ works.

## Level 2

What additional information is needed to solve this problem?
Can you see other
relationships that will
help you find this information?
How can you put your data in graphic form?
What occurs when..?
Does it make sense
to...?
Compare and contrast
$\qquad$ to $\qquad$
What was important about...
What prior
research/formulas
support your conclusions?
How else could you account for...?
Explain how you calculate...
What equation can you write to solve the word problem?

Level 3
Predict what will happen to
$\qquad$ as $\qquad$ is changed.
Using a math principle, how can we find ...?
Describe the events that might occur if... Design a scenario for... Pretend you are...
What would the world be like if...
How can you tell if your answer is reasonable?
What would happen to
$\qquad$
$\qquad$
variable were increased/decreased? How would repeated trials affect your data?
What significance is this formula to the subject you're
learning?
What type of evidence is most compelling to you?

## Costa's Levels of Thinking and Questioning

## Costa's Levels of

Questioning -
Science

## Level 1

What information is given?
What are you being asked to find?
What formula would
you use in this
problem?
What does
$\qquad$ mean?
What is the formula
for...?
List the...
Name the...
Where did...?
What is...?
When did...
Describe in your own
words what $\qquad$
means
What science concepts
does this problem connect to?
Draw a diagram of...
Illustrate how $\qquad$ works.

Level 2
What additional
information is needed
to solve this problem?
Can you see other
relationships that will
help you find this
information?
How can you put your
data in graphic form?
How would you
change your
procedures to get better
results?
What method would you use to...
Compare and contrast
$\qquad$
Which errors most
affected your results?
What were some
sources of variability?
How do your
conclusions support
your hypothesis?
What prior
research/formulas
support your
conclusions?
How else could you account for...?
Explain the concept
of...
Give me an example of...

Level 3
Design a lab to show...
Predict what will
happen to
$\qquad$ as $\qquad$ is changed
Using a science
principle of, how can we find ...?
Describe the events
that might occur if...
Design a scenario for... Pretend you are...
What would the world be like if...
What would happen to
$\qquad$ if $\qquad$
variable were increased/decreased? How would repeated trials affect your data? What significance is this experiment to the subject you're learning?
What type of evidence is most compelling to you?
Do you feel $\qquad$ experiment is ethical?
Are your results biased?

## Costa's Levels of Thinking and Questioning

## Costa's Levels of

Questioning -
English

## Level 1

What information is given?
Locate in the story where...
When did the event take place?
Point to the...
List the...
Name the...
Where did...?
What is...?
Who was/were...?
Illustrate the part of the story that...
Make a map of...
What is the origin of the word
$\qquad$ ?
What events led to
$\qquad$ ?

## Level 2

What would happen to you if..
Would you have done the same thing as...?
What occurs when..?
Compare and contrast
$\qquad$ to $\qquad$
What other ways could be interpreted?
What is the main idea of the story (event)?
What information
supports your
explanation?
What was the message in this piece (event)...
Give me an example of...
Describe in your own
words what $\qquad$ means.
What does $\qquad$
suggest about 's
character?
What lines of the poem
express the poet's
feelings about
$\qquad$
What is the author trying to prove? What evidence does he present?

## Level 3

Design a $\qquad$ to show...
Predict what will happen to $\qquad$ as
$\qquad$ is changed
Write a new ending to the story (event)... Describe the events that might occur if... Add something new on your own that was not in the story...
Pretend you are...
What would the world be like if...
Pretend you are a character in the story. Rewrite the episode from your point of view.
What do you think will happen to $\qquad$ ?
Why?
What is most compelling to you in this $\qquad$ ? Why? Could this story have really happened? Why or why not?
If you were there, would you...
How would you solve this problem in your life?

## Costa's Levels of Thinking and Questioning

## Costa's Levels of <br> Questioning - <br> Social Studies

## Level 1

What information is given?
What are you being asked to find?
When did the event take place?
Point to the...
List the...
Name the...
Where did...?
What is...?
Who was/were...?
Make a map of...

## Level 2

What would happen to you if..
Can you see other relationships that will
help you find this information?
Would you have done the same thing as...?
What occurs when..?
If you were there, would you...
How would you solve this problem in your life?
Compare and contrast
$\qquad$ to $\qquad$
What other ways could
$\qquad$ be interpreted?
What things would you have used to...
What is the main idea of the event?
What information
supports your
explanation?
What was the message in this event...
Explain the concept of...
Give me an example of...
Describe in your own words

## Level 3

Design a $\qquad$ to
show...
Predict what will happen to
$\qquad$ as $\qquad$ is changed
What would it be like to live...
Write a new ending to the event...
Describe the events that might occur if... Pretend you are...
What would the world be like if...
How can you tell if your analysis is reasonable?
What do you think will happen to $\qquad$ ?
Why?
What significance is this event in the global perspective?
What is most compelling to you in this $\qquad$ ? Why?
$\qquad$ is ethical? Why or why not?

## Depth of Knowledge (DOK) Levels




| Level Two Activities | Level Three Activities |
| :--- | :--- |
| Identify and summarize the major <br> events in a narrative. | Support ideas with details and <br> examples. |
| Use context cues to identify the <br> meaning of unfamiliar words. <br> Solve routine multiple-step problems. | Use voice appropriate to the <br> purpose and audience. <br> Identify research questions and <br> design investigations for a <br> scientific problem. |
| Describe the cause/effect of a <br> particular event. | Develop a scientific model for a <br> complex situation. |
| Identify patterns in events or <br> behavior. | Determine the author's purpose <br> and describe how it affects the <br> interpretation of a reading <br> Formulate a routine problem given <br> data and conditions. |
| Organize, represent and interpret <br> data. | Apply a concept in other contexts. |

Level Four Activities
Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions.

Apply mathematical model to illuminate a problem or situation.

Analyze and synthesize information from multiple sources.

Describe and illustrate how common themes are found across texts from different cultures.

Design a mathematical model to inform and solve a practical or abstract situation.

## Appendix B: Cognitive Rigor Matrix/Depth of Knowledge

The Common Core State Standards require high-level cognitive demand, such as asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target in this document, the "depth(s) of knowledge" that the student needs to bring to the item/task has been identified, using the Cognitive Rigor Matrix shown below. This matrix draws from two widely accepted measures to describe cognitive rigor: Bloom's (revised) Taxonomy of Educational Objectives and Webb's Depth-of-Knowledge Levels. The Cognitive Rigor Matrix has been developed to integrate these two models as a strategy for analyzing instruction, for influencing teacher lesson planning, and for designing assessment items and tasks. (To download full article describing the development and uses of the Cognitive Rigor Matrix and other support CRM materials, go to: http://www.nciea.org/publications/cognitiverigorpaper_KH11.pdf)

A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones, \& Walkup, 2009)

| Depth of Thinking (Webb) <br> + Type of Thinking (Revised Bloom) | DOK Level 1 Recall \& Reproduction | DOK Level 2 Basic Skills \& Concepts | DOK Level 3 Strategic Thinking \& Reasoning | DOK Level 4 Extended Thinking |
| :---: | :---: | :---: | :---: | :---: |
| Remember | - Recall conversions, terms, facts |  |  |  |
| Understand | -Evaluate an expression -Locate points on a grid or number on number line <br> -Solve a one-step problem <br> -Represent math relationships in words, pictures, or symbols | - Specify, explain relationships -Make basic inferences or logical predictions from data/observations -Use models /diagrams to explain concepts -Make and explain estimates | -Use concepts to solve nonroutine problems <br> -Use supporting evidence to justify conjectures, generalize, or connect ideas -Explain reasoning when more than one response is possible <br> -Explain phenomena in terms of concepts | -Relate mathematical concepts to other content areas, other domains -Develop generalizations of the results obtained and the strategies used and apply them to new problem situations |
| Apply | -Follow simple procedures <br> -Calculate, measure, apply a rule (e.g., rounding) <br> -Apply algorithm or formula -Solve linear equations -Make conversions | -Select a procedure and perform it <br> -Solve routine problem applying multiple concepts or decision points <br> -Retrieve information to solve a problem <br> -Translate between representations | -Design investigation for a specific purpose or research question <br> - Use reasoning, planning, and supporting evidence -Translate between problem \& symbolic notation when not a direct translation | -Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results |
| Analyze | -Retrieve information from a table or graph to answer a question -Identify a pattern/trend | -Categorize data, figures <br> -Organize, order data <br> -Select appropriate <br>  <br> display data <br> -Interpret data from a <br> simple graph <br> -Extend a pattern | -Compare information within or across data sets or texts -Analyze and draw conclusions from data, citing evidence <br> -Generalize a pattern <br> -Interpret data from complex graph | -Analyze multiple sources of evidence or data sets |
| Evaluate |  |  | -Cite evidence and develop a Iogical argument <br> -Compare/contrast solution methods <br> -Verify reasonableness | -Apply understanding in a novel way, provide argument or justification for the new application |
| Create | - Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept | -Generate conjectures or hypotheses based on observations or prior knowledge and experience | -Develop an alternative solution -Synthesize information within one data set | -Synthesize information across multiple sources or data sets <br> -Design a model to inform and solve a practical or abstract situation |

## Appendix A: Cognitive Rigor Matrix/Depth of Knowledge

The Common Core State Standards require high-level cognitive demand, such as asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target in this document, the "depth(s) of knowledge" that the student needs to bring to the item/task has been identified, using the Cognitive Rigor Matrix shown below. This matrix draws from two widely accepted measures to describe cognitive rigor: Bloom's (revised) Taxonomy of Educational Objectives and Webb's Depth-of-Knowledge Levels. The Cognitive Rigor Matrix has been developed to integrate these two models as a strategy for analyzing instruction, for influencing teacher lesson planning, and for designing assessment items and tasks.
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| :---: | :---: | :---: | :---: | :---: |
| Remember | - Recall, locate basic facts, definitions, details, events |  |  |  |
| Understand | - Select appropriate words for use when intended meaning is clearly evident | - Specify, explain relationships <br> - summarize <br> - identify central ideas | - Explain, generalize, or connect ideas using supporting evidence (quote, text evidence, example...) | - Explain how concepts or ideas specifically relate to other content domains or concepts |
| Apply | - Use language structure (pre/suffix) or word relationships (synonym/antonym) to determine meaning | - Use context to identify word meanings <br> - Obtain and interpret information using text features | - Use concepts to solve non-routine problems | - Devise an approach among many alternatives to research a novel problem |
| Analyze | - Identify the kind of information contained in a graphic, table, visual, etc. | - Compare literary elements, facts, terms, events - Analyze format, organization, \& text structures | - Analyze or interpret author's craft (e.g., literary devices, viewpoint, or potential bias) to critique a text | - Analyze multiple sources or texts - Analyze complex abstract themes |
| Evaluate |  |  | - Cite evidence and develop a logical argument for conjectures based on one text or problem | - Evaluate relevancy, accuracy, \& completeness of information across texts/ sources |
| Create | - Brainstorm ideas, concepts, problems or perspectives related to a topic or concept | -Generate conjectures or hypotheses based on observations or prior knowledge and experience | $\begin{aligned} & \text {-Develop a complex } \\ & \text { model for a given } \\ & \text { situation } \\ & \text {-Develop an alternative } \\ & \text { solution } \end{aligned}$ | -Synthesize information across multiple sources or texts -Articulate a new voice, alternate theme, new knowledge or perspective |

## Reacting to Reading: Drawing Conclusions (I Read/I Think/Therefore)

## What teachers do

## What students do

## Before

- Select text related to a current topic or issue in the course. Create a question or reading prompt to guide the reading (e.g., "How does light enter your eye?" "Describe the games of soccer or football.").
- Prepare a scenario based on the topic or issue. Provide students with information and details about the subject.
- Use a thinking strategy such as "I Read/I Think/Therefore" to demonstrate how to draw a conclusion based on gathered information.
See Tips and Resources on the previous page.
- Provide students with a graphic organizer to record their thinking as they read a coursespecific text. See Student Resource, Template for Drawing Conclusions.
- Provide students with copies of the reading selection and ask them to preview it.
- Set a purpose for reading.
- Use a transparency of the graphic organizer to model for students how to read and record information and inferences. Read the first two or three paragraphs to model the process.


## During

- Ask students in pairs or individually to complete the reading task and the "I Read" and "I Think" columns of the graphic organizer.
- Partners may read, pause, discuss and record the information and their thinking.
- Read the information provided and make inferences based on the information.
- Make a conclusion.
- Observe the teacher's thinking process for drawing a conclusion.
- Preview the text to get ready to read.
- Clarify the purpose for reading (prompt or question).
- Observe how to complete the graphic organizer.
- Read the text, pausing to record important information, and make inferences.


## After

- Review the information gathered in the "I Read" section. Note responses and ask students to account for similarities and differences.
- Compile information on the transparency of the graphic organizer.
- Discuss the students' responses in the "I Think" section. Model how to make inferences, and complete the section on the transparency.
- Review the information and inferences. Ask students to suggest conclusions that can be made based on the information gathered so far. Discuss possible "Therefore" conclusions.
- Model how to make a conclusion based on gathered information.
- Ask students to use this thinking process to read a short passage on the same topic. Ask students to share and compare their conclusions.
- Reread their graphic organizers. Identify similarities and differences among responses.
- Draw a conclusion based on the information and inferences in the chart.
- Compare own conclusion with those of others.
- Apply their learning to a different reading task.


## I Read/l Think/Therefore - Sample Response


#### Abstract

Students are encouraged to use the graphic organizer on the following page to read and respond to a particular text. However, they can also use it to accumulate information about a topic from several sources before drawing a conclusion. For example, students may be investigating the issue of Aboriginal right to self-government, a country's responsibility for its past actions/decisions, or the challenge of diverse cultures working together. They may need to read several different sources to develop a full understanding of the topic or issue.


The text says that Aboriginal selfgovernment was part of the Charlottetown Accord, but was defeated in a national referendum. I think I need more information about why it was defeated.

The text says that the Nisga'a negotiated and signed a treaty for wide-powers of self-government. I
think that this could be a model for other Provinces. I think there still needs to be lots of discussion about economic and political matters.

We started this section with Elijah Harper's opposition to the Meech Lake Accord in 1990. You will remember that the Accord was designed to persuade Quebec to sign the 1982 Canadian Constitution by giving the province special status. Harper opposed the Accord because he believed that Aboriginal Peoples deserved special status, too. With that status, the inherent right to Aboriginal self-government would be recognized. After the defeat of the Meech Lake Accord, the government of Prime Minister Mulroney tried again to revise the Constitution. This time, Aboriginal selfgovernment was included in the agreement, called the Charlottetown Accord, though what self-government involved was not defined. However, this Accord was defeated in a national referendum in 1992.

Since then, Aboriginal Peoples have made some gains. A major one was in 1998 when the federal government issued a Statement of Reconciliation. It stated that government policies had undermined Aboriginal political, economic, and social systems in the past. The federal government apologized for past mistakes and went on to state that

> In renewing our partnership, we must ensure that the mistakes which marked our past relationship are not repeated. The Government of Canada recognizes that policies that sought to assimilate Aboriginal people, women and men, are not the way to build a strong country.

Also in 1998, after 30 years of negotiations, the Nisga'a signed a treaty with British Columbia and the federal government. In 2000 the treaty was officially ratified by Parliament. In this treaty, the Nisga'a were given wide powers of self-government in matters of culture, language, and family life.

The text says that the Meech Lake Accord gave Quebec special status. I think that the Constitution should recognize the unique backgrounds of all of the Provinces and Territories.

The text says that the government apologized for its past mistakes in a Statement of Reconciliation. I think this is a big step forward in trying to bring these two groups together.

## Therefore...

The issue of Aboriginal self-government is a very complex issue. There are still many concerns that have not been addressed in political and economic matters.

THINK LITERACY: Cross-Curricular Approaches, Grades 7-12

Template for Drawing Conclusions

Therefore...

# YOUNG ADULTS HIT BY 'NEWS FATIGUE' 

By KARL RITTER Associated Press Writer<br>STOCKHOLM, Sweden June 2, 2008 (AP)

Young adults experience news fatigue from being inundated by facts and updates and have trouble accessing in-depth stories, according to a study to be unveiled at a global media conference Monday.

The Context-Based Research Group, an ethnographic research firm, found that the news consumption behaviour of younger readers differs profoundly from that of previous generations.

The research project, commissioned by The Associated Press in 2007, analyzed the news consumption patterns of an ethnically diverse group of 18 men and women between the ages of 18 and 34 in six cities in the United States, Britain and India.

It ultimately helped AP design a new model for news delivery to meet the needs of young adults, who are driving the shift from traditional media to digital news, said Jim Kennedy, AP's director of strategic planning.
' 'The real value was that it gave us a lasting model of how news is being consumed in the digital space by young people that we can use to improve our own newsgathering and project development," Kennedy said.

That includes what the AP calls ``1-2-3 filing," starting with a news alert headline for breaking news, followed by a short present-tense story that is usable on the Web and by broadcasters. The third step is to add details and format stories in ways most appropriate for various news platforms.

Editors at the Telegraph in London are following a similar approach and have seen a big jump in traffic at the newspaper's Web site. The study said the Telegraph has adopted the mind-set of a broadcast-news operation to quickly build from headlines to short stories to complete multimedia packages online to boost readership.

The study's purpose was to obtain a deeper and more holistic understanding of the news consumption behavior of younger audiences. The results were scheduled to be presented Monday in a 71-page report to media executives and editors from around the globe at the World Editors Forum in Goteborg, southwestern Sweden.

A key finding was that participants yearned for quality and in-depth reporting but had difficulty immediately accessing such content because they were bombarded by facts and updates in headlines and snippets of news.

The study also found that participants were unable to give full attention to the news because they were almost always simultaneously engaged in other activities, such as reading e-mail. That represents a shift from previous consumption models in which people sat down to watch the evening news or read the morning paper.
``Our observations and analysis identified that consumers' news diets are out of balance due to the over-consumption of facts and headlines," said Robbie Blinkoff, co-founder and head anthropologist at Baltimore, Md.based Context-Based Research Group.

To combat that, the authors recommended that news producers develop easier ways for readers to discover in-depth content and to avoid repetitious updates of breaking news.

The research was conducted in six major metropolitan areas around the globe: Houston, Silicon Valley, Philadelphia and Kansas City in the United States; Brighton, Britain; and Hyderabad, India.

## Properties of Basic Mathematical Operations

Some mathematical operations have properties that can make them easier to work with and can actually save you time.

## Some properties (axioms) of addition

You should know the definition of each of the following properties of addition and how each can be used.

- Closure is when all answers fall into the original set. If you add two even numbers, the answer is still an even number $(2+4=6)$; therefore, the set of even numbers is closed under addition (has closure). If you add two odd numbers, the answer is not an odd number $(3+5=8)$; therefore, the set of odd numbers is not closed under addition (no closure).
- Commutative means that the order does not make any difference in the result.
$2+3=3+2$
$a+b=b+a$
Note: Commutative does not hold for subtraction.

$$
\begin{aligned}
3-1 & \neq 1-3 \\
2 & \neq-2 \\
a-b & \neq b-a
\end{aligned}
$$

- Associative means that the grouping does not make any difference in the result.

$$
\begin{aligned}
& (2+3)+4=2+(3+4) \\
& (a+b)+c=a+(b+c)
\end{aligned}
$$

The grouping has changed (parentheses moved), but the sides are still equal.
Note: Associative does not hold for subtraction.

$$
\begin{aligned}
4-(3-1) & \neq(4-3)-1 \\
4-2 & \neq 1-1 \\
2 & \neq 0 \\
a-(b-c) & \neq(a-b)-c
\end{aligned}
$$

- The identity element for addition is 0 . Any number added to 0 gives the original number.

$$
\begin{aligned}
& 3+0=0+3=3 \\
& a+0=0+a=a
\end{aligned}
$$

- The additive inverse is the opposite (negative) of the number. Any number plus its additive inverse equals 0 (the identity).
$3+(-3)=0$; therefore, 3 and -3 are additive inverses.
$-2+2=0$; therefore, -2 and 2 are additive inverses.
$a+(-a)=0$; therefore, $a$ and $-a$ are additive inverses.


## Some properties (axioms) of multiplication

You should know the definition of each of the following properties of multiplication and how each can be used.

- Closure is when all answers fall into the original set. If you multiply two even numbers, the answer is still an even number ( $2 \times 4=8$ ); therefore, the set of even numbers is closed under multiplication (has closure). If you multiply two odd numbers, the answer is an odd number ( $3 \times 5=15$ ); therefore, the set of odd numbers is closed under multiplication (has closure).
- Commutative means the order does not make any difference.
$2 \times 3=3 \times 2$
$a \times b=b \times a$
Note: Commutative does not hold for division.

$$
\begin{aligned}
2 \div 4 & \neq 4 \div 2 \\
\frac{2}{4} & \neq \frac{4}{2} \\
\frac{1}{2} & \neq 2 \\
a \div b & \neq b \div a
\end{aligned}
$$

- Associative means that the grouping does not make any difference.
$(2 \times 3) \times 4=2 \times(3 \times 4)$
$(a \times b) \times c=a \times(b \times c)$
The grouping has changed (parentheses moved) but the sides are still equal.
Note: Associative does not hold for division.

$$
\begin{aligned}
(8 \div 4) \div 2 & \neq 8 \div(4 \div 2) \\
2 \div 2 & \neq 8 \div 2 \\
1 & \neq 4 \\
(a \div b) \div c & \neq a \div(b \div c)
\end{aligned}
$$

- The identity element for multiplication is 1 . Any number multiplied by 1 gives the original number.

$$
\begin{aligned}
& 3 \times 1=1 \times 3=3 \\
& a \times 1=1 \times a=a
\end{aligned}
$$

- The multiplicative inverse is the reciprocal of the number. Any nonzero number multiplied by its reciprocal equals 1.
$2 \times \frac{1}{2}=1$; therefore, 2 and $\frac{1}{2}$ are multiplicative inverses.
$a \times \frac{1}{a}=1$; therefore, $a$ and $\frac{1}{a}$ are multiplicative inverses (provided a 0 ).


## A property of two operations

The distributive property is the process of passing the number value outside of the parentheses, using multiplication, to the numbers being added or subtracted inside the parentheses. In order to apply the distributive property, it must be multiplication outside the parentheses and either addition or subtraction inside the parentheses.

$$
\begin{array}{rlrl}
2(3+4) & =2(3)+2(4) & 5(12-3) & =5(12)-5(3) \\
2(7) & =6+8 & 5(9) & =60-15 \\
14 & =14 & 45 & =45 \\
a(b+c) & =a(b)+a(c) & a(b-c) & =a(b)-a(c)
\end{array}
$$

Note: You cannot use the distributive property with only one operation.

$$
\begin{aligned}
3(4 \times 5 \times 6) & \neq 3(4) \times 3(5) \times 3(6) \\
3(120) & \neq 12 \times 15 \times 18 \\
360 & \neq 3240 \\
a(b c d) & \neq a(b) \times a(c) \times a(d) \text { or } \\
a(b c d) & \neq(a b)(a c)(a d)
\end{aligned}
$$

## Word Problems Made Easy

1) This is an introduction to understanding word problems.
2) It deals primarily with the translation of word problems into equations.

## The Following Points Outline a General Approach to Word Problems:

1) Read the entire question carefully and get a feel for what is happening. Identify what kind of word problem you're up against.
2) Make a note of exactly what is being asked.
3) Simplify the problem - this is what is usually meant by 'translating the English to Math'. Draw a figure or table. Sometimes a simple illustration makes the problem much easier to approach.
4) It is not always necessary to start from the first line. Invariably, you will find it easier to define what you have been asked for and then work backwards to get the information that is needed to obtain the answer.
5) Use variables ( $a, b, x, y$, etc.) or numbers (100 in case of percentages, any common multiple in case of fractions, etc.) depending on the situation.
6) Use SMART values. Think for a moment and choose the best possible value that would help you reach the solution in the quickest possible time. DO NOT choose values that would serve only to confuse you. Also, remember to make note of what the value you selected stands for.
7) Once you have the equations written down it's time to do the math! This is usually quite simple. Be very careful so as not to make any silly mistakes in calculations.
8) Lastly, after solving, cross check to see that the answer you have obtained corresponds to what was asked. The makers of these GMAT questions love to trick students who don't pay careful attention to what is being asked. For example, if the question asks you to find 'what fraction of the remaining...' you can be pretty sure one of the answer choices will have a value corresponding to 'what fraction of the total...'

## Translating Word Problems

These are a few common English to Math translations that will help you break down word problems. My recommendation is to refer to them only in the initial phases of study. With practice, decoding a word problem should come naturally. If, on test day, you still have to try and remember what the math translations to some English term is, you haven't practiced enough!

ADDITION : increased by ; more than ; combined ; together ; total of ; sum ; added to ; and ; plus
SUBTRACTION : decreased by ; minus ; less ; difference between/of ; less than ; fewer than ; minus ; subtracted from

MULTIPLICATION : of ; times ; multiplied by ; product of ; increased/decreased by a factor of (this
type can involve both addition or subtraction and multiplication!)
DIVISION : per ; out of ; ratio of ; quotient of ; percent (divide by 100) ; divided by ; each
EQUALS : is ; are ; was ; were ; will be ; gives ; yields ; sold for ; has ; costs ; adds up to ; the same as; as much as

VARIABLE or VALUE : a number ; how much ; how many ; what

## Some Tricky Forms :

## 'per' means 'divided by'

Jack drove at a speed of 40 miles per hour OR 40 miles/hour.
'a' sometimes means 'divided by'
Jack bought twenty-four eggs for $\$ 3$ a dozen.

## 'less than'

In English, the 'less than' construction is reverse of what it is in math.
For example, ' 3 less than $x$ ' means ' $x-3$ ' NOT ' $3-x$ '
Similarly, if the question says 'Jack's age is 3 less than that of Jill', it means that Jacks age is 'Jill's age - $3^{\prime}$.

## The 'how much is left' construction

Sometimes, the question will give you a total amount that is made up of a number of smaller amounts of unspecified sizes. In this case, just assign a variable to the unknown amounts and the remaining amount will be what is left after deducting this named amount from the total.
Consider the following:
A hundred-pound order of animal feed was filled by mixing products from Bins A, B and C, and that twice as much was added from Bin C as from Bin A.
Let "a" stand for the amount from Bin A. Then the amount from Bin C was " 2 a ", and the amount taken from Bin B was the remaining portion of the hundred pounds: $100-\mathrm{a}-2 \mathrm{a}$.

In the following cases, order is important:

## 'quotient/ratio of' construction

If a problems says 'the ratio of $x$ and $y$ ', it means ' $x$ divided by $y$ ' NOT ' $y$ divided by $x$ '

## 'difference between/of' construction

If the problem says 'the difference of $x$ and $y$ ' it means ' $x-y$ '

Now that we have seen how it is possible, in theory, to break down word problems, lets go through a few simple examples to see how we can apply this knowledge.

## Example 1.

The length of a rectangular garden is 2 meters more than its width. Express its length in terms of its width.

## Solution 1.

Key words: more than (implies addition); is (implies equal to)
Thus, the phrase 'length is 2 more than width' becomes:
Length $=2+$ width

## Example 2.

The length of a rectangular garden is 2 meters less than its width. Express its length in terms of its width.

## Solution 2.

Key words: less than (implies subtraction but in reverse order); is (implies equal to) Thus, the phrase 'length is 2 less than width' becomes:
Length = width - 2

## Example 3.

The length of a rectangular garden is 2 times its width. Express its length in terms of its width.

## Solution 3.

Key words: times (implies multiplication); is (implies equal to)
Thus, the phrase 'length is 2 times width' becomes:
Length $=2 *$ width

## Example 4.

The ratio of the length of a rectangular garden to its width is 2 . Express its length in terms of its width.

## Solution 4.

Key words: ratio of (implies division); is (implies equal to)
Thus, the phrase 'ratio of length to width is 2 ' becomes:
Length/width $=2 \rightarrow$ Length $=2^{*}$ width

## Example 5.

The length of a rectangular garden surrounded by a walkway is twice its width. If difference between the length and width of just the rectangular garden is 10 meters, what will be the width of the walkway if just the garden has width 6 meters?

## Solution 5.

Ok this one has more words than the previous examples, but don't worry, lets break it down and see how simple it becomes.
Key words: and (implies addition); twice (implies multiplication); difference between (implies subtraction where order is important); what (implies variable); is, will be (imply equal to)

Since this is a slightly more complicated problem, let us first define what we want.
'What will be the width of the walkway' implies that we should assign a variable for width of the walkway and find its value.

Thus, let width of the walkway be ' $x$ '.

Now, in order to find the width of walkway, we need to have some relation between the total length/width of the rectangular garden + walkway and the length/width of just the garden.

Notice here that if we assign a variables to the width and length of either garden+walkway or just garden, we can express every thing in terms of just these variables.

So, let length of the garden+walkway $=L$
And width of garden+walkway $=\mathrm{W}$
Thus length of just garden $=L-2 x$
Width of just garden $=W-2 x$
Note: Remember that the walkway completely surrounds the garden. Thus its width will have to be accounted for twice in both the total length and total width.

Now let's see what the question gives us.
‘Garden with width 6 meters' translates to:
Width of garden $=6$
$W-2 x=6$
Thus, if we know $W$ we can find $x$.
'Length of a rectangular garden surrounded by walkway is twice its width' translates to:
Length of garden + length of walkway $=2 *($ width of garden + width of walkway)
$L=2 * W$
'Difference between the length and width of just the rectangular garden is 10 meters' translates to:
Length of garden - width of garden $=10$
$(L-2 x)-(W-2 x)=10$
$L-W=10$
Now, since we have two equations and two variables ( $L$ and W), we can find their values. Solving them we get: $L=20$ and $W=10$.

Thus, since we know the value of $W$, we can calculate ' $x$ '
$10-2 x=6$
$2 x=4$
$x=2$

## Thus, the width of the walkway is 2 meters.

Last edited by sriharimurthy on Thu Dec 03, 2009 9:56 am, edited 4 times in total.
Downloaded 11-27-11: http://gmatclub.com/forum/word-problems-made-easy-87346.html

Rayburn Mathematics Rubric

|  | Novice | Apprentice | Practitioner | Expert |
| :---: | :---: | :---: | :---: | :---: |
| Strategies, Reasoning \& Procedures | - Lacks understanding of strategies and procedures <br> - Work does not lead to correct solution | - Strategies and procedure evident with support <br> - Minor errors are present | - Strategies and procedures clearly evident <br> - Lacks connections between prior learning and new learning | - Strategies and procedures clearly evident <br> - Connections are made between prior learning and new learning |

## Scientific Thinking Rubric

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Critical Thinking | - Often requires help in order to critically examine information <br> - Support, if provided, is irrelevant or based on personal opinion, disregarding scientific principles and knowledge | - Requires teacher prompting to examine information using scientific principles <br> - Generalized supporting example is somewhat relevant <br> - May use some scientific knowledge | - Mostly works independently to examine information using scientific principles and knowledge <br> - 1 or 2 somewhat generalized but relevant examples provided to support or refute ideas | - Independently examines information using scientific principles and knowledge <br> - Provides 2 or 3 relevant examples to support or refute ideas | - Independently examines information using scientific principles and knowledge <br> - Will provide 3 or 4 specific and relevant examples to support or refute ideas |
| Problem Solving | - Is not able to recognize or understand problem without teacher instruction <br> - Is not able to use scientific process without teacher help | - Often requires teacher prompts to recognize and understand problem <br> - Often requires teacher help to begin using scientific process | - Generally able to recognize and understand problem <br> - Able to use scientific process to attack problem <br> - Will on occasion get help from peers to use scientific process to seek solutions | - Independently recognizes and understands problems <br> - Uses scientific process to seek solutions <br> - Will sometimes develop new questions | - Independently recognizes and understands problems <br> - Readily uses scientific process to seek solutions <br> - Will often develop new, original questions |
| Reflection upon Scientific Ideas and Principles | - Requires teacher help to examine ideas <br> - Disregards the ideas of others | - Requires teacher prompts to examine ideas <br> - Has difficulty accepting the ideas of others | - Will generally reflect on ideas <br> - There may be some gaps in logic <br> - Will accept other ideas | - Will, on most occasions, reflect on ideas <br> - Logic is generally sound <br> - Recognizes value of other ideas | - Will consistently reflect on ideas <br> - Logic is sound and provided with sufficient detail <br> - Recognizes value of other ideas <br> - Will often help others to evaluate ideas |
| Creative Thinking | - Has difficulty seeing paths to possible solutions <br> - Often requires teacher significant help to recognize connections in ideas | - Generally will follow established path to possible solutions <br> - On occasion becomes sidetracked, losing path <br> - Requires teacher prompts to recognize connections among ideas | - Will generally generate a logical path to a possible solution <br> - May require some help to recognize connections between own ideas and those of others | - Generally recognizes 1 or 2 paths for possible solutions <br> - Is capable of connecting own ideas with those of others to find solutions to problems | - Consistently sees several paths for possible solutions <br> - Adapts and connects own ideas and those of others to develop unique questions and solutions to problems |

Rubric for Critical Response to Reading

| Criteria | Level 1 | Level 2 | Level 3 | Level 4 | Level 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Comprehension of Text | - Has difficulty clearly stating main point of simple reading material <br> - Confuses main and subordinate points <br> - Keys from an insignificant element <br> - Misinterprets main ideas and themes of reading | - States main point of simple reading material <br> - Misses subordinate points or gives irrelevant details <br> - Comprehension is founded in plot summary | - Clearly states main point of simple and some complex readings <br> - Gives some relevant support and subordinate details <br> - Limited judgments confirm common or obvious views of text | - Clearly identifies main idea in complex and some sophisticated readings <br> - Provides clear support for main points and notes minor points <br> - Sound judgments consider multiple possibilities with some hesitancies | - Clearly and succinctly identifies main idea in complex and sophisticated readings <br> - Sees complexity and nuances of sophisticated readings <br> - Provides clear and succinct interpretations <br> - Attends to significant themes and questions with some objectivity |
| Own Viewpoint and Commentary | - Own viewpoint is unclear, stereotypical or derivative or given as though from the reading <br> - Ideas are simplistic or contradictory | - Own viewpoint is stated and separated from that in reading material <br> - Ideas are somewhat clear <br> - Common themes are used | - Own viewpoint is clear and supported <br> - Ideas are logical and on-topic <br> - Makes a connection from and beyond personal experience | - Own viewpoint is clear and provided with solid support <br> - Ideas are wellconsidered, logical and on-topic <br> - Makes multiple connections from and beyond personal experience | - Offers substantial and varied support or insightful viewpoint <br> - Insightful and well-considered ideas or addresses significant themes <br> - Makes connections with personal experience and ties these also to some abstract principles/ ideas |
| Presentation of Viewpoint | - Focus is confusing. <br> - Voice is stereotyped or copied <br> - Not clear to whom the piece is directed <br> - Organization is confused and confusing to the reader/listener | - Focus is present, though may meander. <br> - Voice is unclear or varies without purpose <br> - Audience identified only generally as teacher <br> - Organization wanders with irrelevancies, repetitions and/ or gaps | - Focus is generally clear and mostly appropriate. <br> - Some enthusiasm in voice/tone interests the readers. <br> - Audience is mostly established <br> - A clear, if formulaic, organizational pattern assists the reader/listener | - Focus is clear and appropriate. <br> - Voice and tone contribute to interest and understanding. <br> - Audience established early and clearly <br> - A clear organizational pattern is assisted by smooth transitions and effective sequencing of information and opinion | - Strong focus provides a central organizing idea to response. <br> - Voice and tone contribute to interest, clarity and understanding. <br> - Audience established and maintained easily <br> - Organization is clear and imaginative |

Sandra Falconer Pace 2005

## Rubric for Group Discussion in Social Studies

| Criterion | $\mathbf{5}$ | $\mathbf{3}$ | 1 |
| :--- | :--- | :--- | :--- |
| Participation | Actively participates, paraphrases; <br> encourages others; is patient; is <br> enthusiastic | Shares some ideas; requires <br> encouragement; sometimes needs <br> redirection; may need prompting. | Off-task; distracting; inappropriate <br> responses or contributions. |
| Critical Thinking | Clearly identifies problem; considers <br> others' viewpoints; formulates <br> conclusions, makes comparisons and <br> contrasts; applies extended learning. | Identifies the problem through <br> restating main points, showing a <br> general understanding; 1 some/limited <br> application of ideas. | Demonstrates little comprehension of <br> the problem; comes to hasty <br> conclusions; little or no consideration <br> of others' viewpoints. |
| Communication of <br> Ideas | Has a well-defined position; uses <br> questioning; paraphrasing and <br> clarifying; uses positive body <br> language (eye contact; posture; <br> voice); supports others and is <br> convincing | Demonstrates ideas in a general way; <br> limited paraphrasing and questioning; <br> passive interaction. | Vague positions; judgmental; <br> confrontational; shares own point of <br> view in a negative fashion or is non- <br> committal/withdrawn. |
| Use of Knowledge | Shares in-depth knowledge; applies <br> knowledge to past; present and <br> future issues. | Shares a general knowledge of the <br> issue. | Refrains from contributing any <br> knowledge or denigrates others' <br> knowledge. |


| Academic | D | C | B | A |
| :---: | :--- | :--- | :--- | :--- |
| Effort | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | 4 |

Lin Kuzmich is an educational consultant and bestselling author from Loveland, Colorado. She served Thompson School District in several roles as the Deputy Superintendent, Executive Director of Secondary and Elementary Instruction, Director of Professional Development and she was a building principal for nine years. Lin's school was named a 2000 winner of the John R. Irwin Award for Academic Excellence and Improvement. In addition, for the past decade she was involved in staff development through several universities and the Tointon Institute for Educational Change. Lin served as an Adjunct Professor and Instructor at Colorado State University and University of Northern Colorado in the Principal Preparation Programs. She is a Senior Consultant for the International Center for Leadership in Education. Lin also provides training and consulting to school districts around the country and presents at numerous national and international conferences. Lin Kuzmich can be reached at 970-669-2290 (home/office) 970-203-4176 (cell) or kuzenergy@gmail.com and her website is www.KuzmichConsulting.com

Lin's additional experience includes: Assistant Director of Special Education (1988-1991); Vision Specialist and Reading Teacher for Thompson School District (1979-1988). She also taught high school reading, high school and middle school English/Language Arts, K-12 special education and $4^{\text {th }}-6^{\text {th }}$ grades for Denver Public Schools (1974-79). Lin earned the Teacher of the Year Award for Denver Public Schools in 1979 and was Northern Colorado Principal of the Year in 2000 for Colorado Association of School Executives.

Lin currently works with schools and districts across the country that are struggling to meet the needs of diverse learners, the requirements of AYP and the changing educational practices needed for the future success of our students. Lin's work with schools improves achievement results for students and increases the capacity of staff. Lin is passionate about helping educators prepare today's students for a successful future.

## Lin's Publications:

- Stretch Learning Handbook- With Units and Strategies Aligned to Common Core State Standards (2011) International Center for Leadership in Education
- "Manage the Molehill Before It Becomes a Mountain: Keeping Parent Interactions Productive for Students" in Leadership for Family and Community Involvement Edited by Cole, Blankstein and Houston for the Soul of Leadership Series (2010) Corwin Press
- Stretch Learning: Rigor and Relevance for an Unpredictable World (2010) International Center for Leadership in Education. (Multi-Media Kit)
- Student Team That Get Results: Teaching Tools for the Differentiated Classroom (2010) Corwin Press, co-author Gayle Gregory.
- "Test Preparation Strategies that Have High and Quick Payoff," (March 2010) Successful Practices Network Monthly Online Publication
- "Ensuring Access through Differentiated Instruction" in The Special EDge, Vol. 21, Num. 3 Summer 2008, coauthored with Dr. Willard Daggett
- Redefining Literacy in Grades 7-12: Strategies for Document, Technological and Quantitative Literacy (May 2007) International Center for Leadership in Education. (Multi-Media Kit)
- Teacher Teams that Get Results: 61 Strategies for Sustaining and renewing Professional Learning Communities (January 2007) Corwin Press, co-author Gayle Gregory. (Bestseller)
- "Redefining Literacy for the $21^{\text {st }}$ Century," (2006a) Successful Practices Network Monthly Online Publication
- "Tips for Credit Recovery Programs," (December 2006b) Successful Practices Network Monthly Online Publication
- Differentiated Literacy Strategies for Student Growth Grades 7-12, (2005b), Corwin Press, co-author Gayle Gregory. (Bestseller)
- Differentiated Literacy Strategies for Student Growth Grades K-6, (2005a) Corwin Press, co-author Gayle Gregory. (Bestseller)
- Data Driven Differentiation in the Standards Based Classroom, (2004) Corwin Press, co-author Gayle Gregory. (Bestseller)
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